

Double Integrals

1. Write a double integral $\iint_{\mathcal{R}} f(x, y) dA$ which gives the volume of the top half of a solid ball of radius 5. (You need to specify a function $f(x, y)$ as well as a region \mathcal{R} .)

2. (a) If \mathcal{R} is any region in the plane (\mathbb{R}^2), what does the double integral $\iint_{\mathcal{R}} 1 dA$ represent? Why?

- (b) Suppose the shape of a flat plate is described as a region \mathcal{R} in the plane, and $f(x, y)$ gives the density of the plate at the point (x, y) in kilograms per square meter. What does the double integral $\iint_{\mathcal{R}} f(x, y) dA$ represent? Why?

3. If \mathcal{R} is the rectangle $[1, 2] \times [3, 4]$, compute the double integral $\iint_{\mathcal{R}} 6x^2y dA$.

4. If \mathcal{R} is the rectangle $[0, 1] \times [-1, 2]$, compute the double integral $\iint_{\mathcal{R}} 2ye^x \, dA$.

5. Find the volume of the solid that lies under $z = x^2 + y^2$ and above the square $0 \leq x \leq 2$, $-1 \leq y \leq 1$.

6. Find the volume of the solid enclosed by the surfaces $z = 4 - x^2 - y^2$, $z = x^2 + 2y^2 - 2$, $x = -1$, $x = 1$, $y = -1$, and $y = 1$.